B_c meson production at the Tevatron Revisited

Kingman Cheung

Department of Physics, University of California, Davis, CA 95616 USA

Abstract

CDF recently measured the quantity $\frac{\sigma(B_c^+)}{\sigma(B^+)}\frac{\mathrm{BR}(B_c^+\to J/\psi\ell^+\nu)}{\mathrm{BR}(B^+\to J/\psi K^+)}$, from which we determine the ratio $\frac{\sigma(B_c^+)}{\sigma(\bar b)}$ to be $(2.08^{+1.06}_{-0.95})\times 10^{-3}$. In this note, we show that the ratio $\frac{\sigma(B_c^+)}{\sigma(\bar b)}$ obtained by dividing the $\sigma(B_c^+)$ by the leading order $\sigma(\bar b)$ is consistent with this derived CDF measurement. We calculate the cross section $\sigma(B_c^+)$ using the perturbative QCD fragmentation functions of Braaten, Cheung, and Yuan and the corresponding induced gluon fragmentation functions, with the charm-quark mass m_c as a parameter. We also estimate the parameter m_c from the CDF data and then predict the production rate at RunII.

1.

CDF [1] and LEP Collaborations [2] recently published their results in search for the final heavy-heavy quark bound state – charmed-beauty meson (B_c). The most impressive is the result by CDF, which established a signal of 4.8σ (from a null hypothesis), using the semi-leptonic decay channels of the B_c meson, $B_c^+\to J/\psi\ell^+\nu$, with $\ell=e,\mu$. CDF measured the ratio

$$\mathcal{R}\equiv\frac{\sigma(B_c^+)}{\sigma(B^+)}\frac{\mathrm{BR}(B_c^+\to J/\psi\ell^+\nu)}{\mathrm{BR}(B^+\to J/\psi K^+)}=0.132^{+0.041}_{-0.037}\,(\mathrm{stat.})\ \pm 0.031\,(\mathrm{syst.})\ ^{+0.032}_{-0.020}\,(\mathrm{lifetime})\ , \tag{1}$$

where in $\mathrm{BR}(B_c^+\to J/\psi\ell^+\nu)$ the branching ratios for e and μ are assumed equal, and the last error comes from the error in the measurement of the B_c lifetime. Based on the following data from Particle Data Book [3]

$$\mathrm{BR}(B^+\to J/\psi K^+)=(9.9\pm 1.0)\times 10^{-4}\ ,\quad \frac{\sigma(B^+)}{\sigma(\bar b)}=0.397^{+0.018}_{-0.022}\ , \tag{2}$$

and a theoretical calculation [4]

$$\mathrm{BR}(B_c^+\to J/\psi\ell^+\nu)=2.5\pm 0.5\% \tag{3}$$

we are able to deduce this ratio

$$\frac{\sigma(B_c^+)}{\sigma(\bar b)}=\frac{\mathrm{BR}(B^+\to J/\psi K^+)}{\mathrm{BR}(B_c^+\to J/\psi\ell^+\nu)}\,\frac{\sigma(B^+)}{\sigma(\bar b)}\times\mathcal{R}=\left(2.08^{+1.06}_{-0.95}\right)\times 10^{-3}\ , \tag{4}$$

where the error is obtained by adding the relative errors in quadrature. Note that the ratio $\frac{\sigma(B^+)}{\sigma(\bar b)}$ in Eq. (2) quoted in Particle Data Book represents the fraction of $\bar b$ that hadronizes into a B^+ meson, which was measured at LEP. This fraction is, to a good approximation, independent of p_T cuts. Thus the ratio $\frac{\sigma(B_c^+)}{\sigma(\bar b)}$ that we obtained in Eq. (4) represents the ratio of the cross section of B_c^+ to the cross section of $\bar b$ under the same p_T cut as the CDF measurement $\mathcal{R}$. This ratio has no direct implication that B_c^+ meson is produced directly from $\bar b$, which is in contrast to B^+ meson that B^+ meson is, in general, assumed coming from the fragmentation of $\bar b$.

The purpose of this note is to verify that the ratio in Eq. (4) is consistent with $\sigma(B_c^+)$ calculated using the perturbation QCD fragmentation functions for $\bar{b} \to B_c^+$ [5, 6, 7, 8] and the corresponding induced gluon fragmentation functions, as well as using the leading order (LO) b-quark production. We shall also obtain the range of the parameters involved in the fragmentation functions. Once we obtain the parameters we can then predict the production rate for the RunII at the Tevatron, where much higher statistics can be accumulated.

The fragmentation approach employed here is different from the full tree-level α_s^4 calculation [9]. Comparison between these two approaches were made in some of the papers in Refs. [9]. Basically, the fragmentation approach gives a reasonable approximation to the full tree-level calculation as long as $p_T > 2M_{B_c}$. Nevertheless, one disadvantage of the full calculation is that higher order effects cannot be easily included unless the NLO calculation is performed. On the other hand, using the fragmentation approach some important higher order effects can be included, namely, the contribution from gluon fragmentation and the contribution from higher orbital states below the BD threshold. We shall show that including these contributions we can easily account for the ratio in Eq. (4) without employing extreme parameters.

2.

In this section, we remind the readers about the importance of the induced gluon fragmentation. The gluon fragmentation function for $g \to B_c^+$ at the initial scale (heavy quark scale) is $O(\alpha_s)$ smaller than the heavy quark fragmentation function for $\bar{b} \to B_c^+$. Thus, the main source of gluon fragmentation comes from the Altarelli-Parisi evolution of the heavy quark fragmentation function. The Altarelli-Parisi evolution equations for the fragmentation functions are

$$\mu \frac{\partial}{\partial \mu} D_{\bar{b}\to H}(z,\mu) = \int_z^1 \frac{dy}{y} P_{\bar{b}\to\bar{b}}(z/y,\mu)\, D_{\bar{b}\to H}(y,\mu) + \int_z^1 \frac{dy}{y} P_{\bar{b}\to g}(z/y,\mu)\, D_{g\to H}(y,\mu) \tag{5}$$

$$\mu \frac{\partial}{\partial \mu} D_{g\to H}(z,\mu) = \int_z^1 \frac{dy}{y} P_{g\to\bar{b}}(z/y,\mu)\, D_{\bar{b}\to H}(y,\mu) + \int_z^1 \frac{dy}{y} P_{g\to g}(z/y,\mu)\, D_{g\to H}(y,\mu) \tag{6}$$

where H denotes any $(\bar{b}c)$ states, and $P_{i\to j}$ are the usual Altarelli-Parisi splitting functions. The initial scale heavy quark and gluon fragmentation functions are [5]

$$\begin{aligned} D_{\bar{b}\to\bar{b}c(n^1S_0)}(z,\mu_0) &= \frac{2\alpha_s(2m_c)^2|R_{nS}(0)|^2}{81\pi m_c^3}\,\frac{rz(1-z)^2}{(1-\bar{r}z)^6}\,\Bigg[6 - 18(1-2r)z \\ &+ (21-74r+68r^2)z^2 - 2\bar{r}(6-19r+18r^2)z^3 + 3\bar{r}^2(1-2r+2r^2)z^4\Bigg]\,, \end{aligned} \tag{7}$$

$$\begin{aligned} D_{\bar{b}\to\bar{b}c(n^3S_1)}(z,\mu_0) &= \frac{2\alpha_s(2m_c)^2|R_{nS}(0)|^2}{27\pi m_c^3}\,\frac{rz(1-z)^2}{(1-\bar{r}z)^6}\,\Bigg[2 - 2(3-2r)z \\ &+ 3(3-2r+4r^2)z^2 - 2\bar{r}(4-r+2r^2)z^3 + \bar{r}^2(3-2r+2r^2)z^4\Bigg]\,, \end{aligned} \tag{8}$$

$$D_{g\to B_c}(z,\mu) = D_{g\to B_c^*}(z,\mu) = 0 \quad \text{for} \quad \mu \le 2(m_b+m_c)\,, \tag{9}$$

where $r = m_c/(m_b + m_c)$, $\bar{r} = 1 - r$, $\mu_0 = m_b + 2m_c$, and $R(0)$ is the radial wavefunction at the origin. They are the initial boundary conditions to the evolution equations in Eqs. (5)–(6). Here we only give the S-wave fragmentation functions, which contribute dominantly to B_c production, the P-wave fragmentation functions can be found in Ref. [7]. Nevertheless, P-wave fragmentation functions contribute only at 10% level to the total B_c production. The less determined parameters in the above functions are $|R(0)|$ and m_c. The value for $R(0)$ can be determined in a potential-model calculation [10]. In Ref. [10], $|R_{1S}(0)|^2$ ranges from 1.5 to 1.7 GeV^3 (the extreme value of 3.2 GeV^3 is not used here.) The fixed input parameters of our present calculation are tabulated in Table 1, while m_c is chosen as a variable parameter in our calculation, because the fragmentation function is very sensitive to m_c, which appears as m_c^3 in the denominator: see Eqs. (7)–(8). Overall, we include all $n = 1$ S-wave and P-wave, and $n = 2$ S-wave states, which are below the BD threshold [10], in our calculation.

	$n = 1$	$n = 2$
m_b	4.9 GeV	4.9 GeV
$R_{nS}(0)$	1.28 GeV$^{3/2}$	0.99 GeV$^{3/2}$
H_1	10 MeV	-
$H_8'(m)$	1.3 MeV	-
$\cos\theta_{\rm mix}$	0.999	-

Table 1: Input parameters to the perturbative QCD fragmentation functions for $n = 1$ and $n = 2$. $H_1, H_8'(m), \cos\theta_{\rm mix}$ are parameters for P-wave states, see Ref. [7].

3.

We are ready to compute the ratio $\sigma(B_c^+)/\sigma(\bar{b})$ with $\sigma(B_c^+)$ calculated by the fragmentation approach and $\sigma(\bar{b})$ by the LO calculation. The "improved" tree-level cross section for B_c^+ is given by

$$\sigma(B_c^+) = \sum_{ij} \int dx_1 dx_2 dz\ f_i(x_1) f_j(x_2) \left[\hat{\sigma}(ij \to \bar{b}X, \mu)\ D_{\bar{b}\to B_c^+}(z,\mu) + \hat{\sigma}(ij \to gX, \mu)\ D_{g\to B_c^+}(z,\mu) \right], \quad (10)$$

where μ is the factorization scale and is chosen to be $\mu = \mu_T \equiv \sqrt{p_{T_{\bar{b}}}^2 + m_b^2}$. We called this the improved cross section because it includes higher order corrections from gluon fragmentation. For $\bar{b}$ cross section we use the LO calculation. When we calculate the ratio of cross sections, the dependence on factorization scale, higher-order QCD corrections, parton distribution functions, and m_b are substantially reduced. We anticipate the ratio $\sigma(B_c^+)/\sigma(\bar{b})$ calculated at tree-level is reasonably accurate without a NLO calculation, providing that the present error on B_c production is very large.

We show the ratio $\sigma(B_c^+)/\sigma(\bar{b})$ versus $p_{T_{\rm min}}(\bar{b})$ for $m_c = 1.2 - 1.7$ GeV in Fig. 1. We note that this ratio increases with $p_{T_{\rm min}}(\bar{b})$, due to the induced gluon fragmentation contribution. When $p_{T_{\rm min}}(\bar{b})$ increases, the scale of the fragmentation function rises and, therefore, the induced gluon fragmentation function also increases. If we did not include the induced gluon fragmentation contribution, the ratio $\sigma(B_c^+)/\sigma(\bar{b})$ would

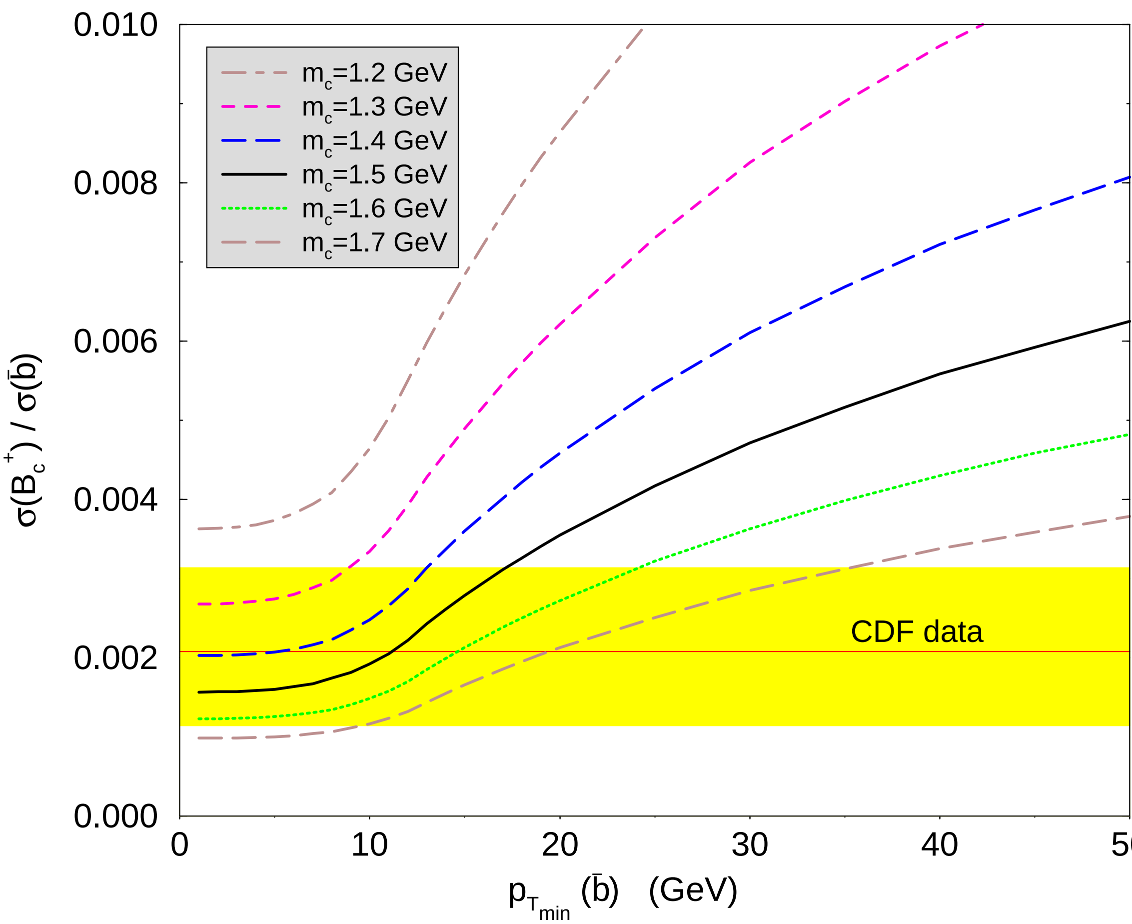

Figure 1: The ratio of $\sigma(B_c^+)/\sigma(\bar{b})$ versus $p_{T_{\min}}$ cut on $\bar{b}$, calculated by fragmentation approach at the Tevatron: $\sqrt{s} = 1.8$ TeV. A rapidity cut of $|y| < 1$ is imposed. The shaded band is the data in Eq. (4), which is derived from the CDF data in Eq. (1).

have been a constant, giving rise to a horizontal line coincide with the lower part of the corresponding curve in Fig. 1. Although the gluon fragmentation probability is much smaller than the $\bar{b}$ fragmentation, the production by gluon fragmentation turns out not negligible, because the amplitude squared of the most important subprocess $gg \to gg$ is more than an order of magnitude larger than that of $gg \to b\bar{b}$ [8]. Figure 1 also shows the sensitivity to m_c.

We put the band of $\sigma(B_c^+)/\sigma(\bar{b})$ given by Eq. (4) onto Fig. 1. We note that the CDF data in Eq. (1) is for B_c^+ and B^+ with $p_T > 6.0$ GeV and $|y| < 1$ [1]. We have to convert this p_T requirement on B^+ and B_c^+ to p_T requirement on $\bar{b}$, because the fragmentation spectrum of $\bar{b}$ is not monochromatic. The average momentum fraction $\langle z \rangle$ for fragmentation of $\bar{b}$ into B^+ and B_c^+ is about $0.7 - 0.8$ at the scale $\mu \approx 8 - 10$ GeV. Hence, the p_T requirement on $\bar{b}$ becomes $8 - 9$ GeV. From Fig. 1 at around $p_{T_{\min}}(\bar{b}) = 8 - 9$ GeV, the shaded CDF band gives

$$m_c \simeq 1.3 - 1.7 \text{ GeV} , \tag{11}$$

with the central value at about 1.45 GeV. Since the error of the ratio in Eq. (4) is large, the range of m_c

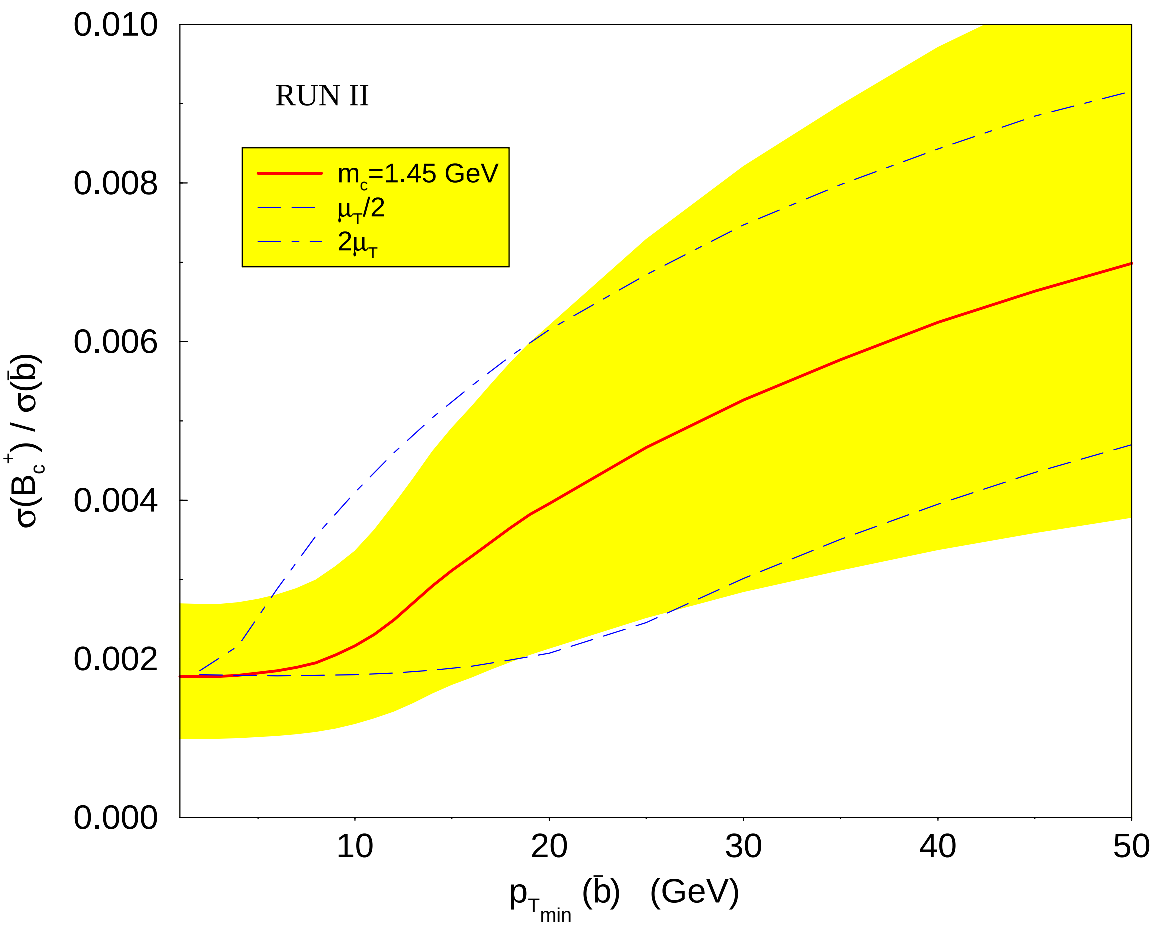

Figure 2: The ratio of $\sigma(B_c^+)/\sigma(\bar{b})$ versus $p_{T_{\rm min}}$ cut on $\bar{b}$ calculated by fragmentation approach at Run II: $\sqrt{s} = 2$ TeV. The shaded region corresponds to $m_c \simeq 1.3 - 1.7$ GeV with the solid line at $m_c = 1.45$ GeV. A rapidity cut of $|y| < 1$ is imposed. The factorization scale $\mu = \mu_T \equiv \sqrt{p_{T_{\bar{b}}}^2 + m_b^2}$ for the solid line, $\mu = \mu_T/2$ for the dashed, and $\mu = 2\mu_T$ for the dot-dashed.

obtained in Eq. (11) is also very wide.

4.

Run II at the Tevatron will be at $\sqrt{s} = 2$ TeV with a nominal accumulated luminosity of 2 fb^{-1}. The prediction of $\sigma(B_c^+)/\sigma(\bar{b})$ for the range of $m_c \simeq 1.3 - 1.7$ GeV obtained above in Eq. (11) is given in Fig. 2 (shaded region) with the solid line for $m_c = 1.45$ GeV. It appears that the ratio predicted at $\sqrt{s} = 2$ TeV is about the same as at $\sqrt{s} = 1.8$ TeV.

Finally, we also demonstrate the dependence of the ratio on the factorization scale, which appears in the running α_s, the parton distribution functions, and the fragmentation functions. In Fig. 2, the solid line is for the original choice of $\mu = \mu_T \equiv \sqrt{p_{T_{\bar{b}}}^2 + m_b^2}$, while the dashed is for $\mu = \mu_T/2$ and the dot-dashed for $\mu = 2\mu_T$. The dependence on the scale is smaller than the dependence on m_c.

To summarize we have obtained the ratio $\sigma(B_c^+)/\sigma(\bar{b}) = \left(2.08^{+1.06}_{-0.95}\right) \times 10^{-3}$ from the CDF data in Eq. (1). We have also verified that the prediction by the perturbative QCD fragmentation approach is consistent with the CDF data, with $m_c \simeq 1.3 - 1.7$ GeV and the central value at 1.45 GeV. The prediction of the ratio at Run II is very similar to that at Run I.

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